ACADEMO
Revista de Investigación en Ciencias Sociales y Humanidades
ISSN 2414-8938. Vol. 12 Nro. 2, 2025.

ORIGINAL ARTICLE

https://doi.org/10.30545/academo.2025.n2.1181

The Benford´s Law (first digit Law) in the public procurement of fuels and lubricants in Paraguay. Period 2014 - 2023

La Ley de Benford (Ley del Primer Dígito) en las compras públicas de combustibles y lubricantes en Paraguay. Período 2014 - 2023

Luis Darío Ríos Barrios[1]

[1] Universidad Católica “Nuestra Señora de la Asunción” Campus Guairá, Facultad de Ciencias Contables, Administrativas y Económicas. Guairá, Paraguay.

Abstract

This research project arises from Paraguay's need to efficiently use resources in public procurement of fuels and lubricants. This is crucial because Paraguay, as a landlocked country and net importer of fuels, requires significant sums for their acquisition. These resources are then utilized by various public institutions such as health and internal security for their services to function properly. The methodology employed is Benford's Law, also known as the First Digit Law, which despite lacking a theoretical foundation, holds strategic empirical importance. The main objective of this study is to identify potential anomalies or irregularities in public procurement of fuels and lubricants in Paraguay from 2014 to 2023. Results indicate anomalies in digit frequency across three analysed variables: awarded amounts, number of contracts awarded or bidders awarded per process, and number of bidders per process. The study concludes that it is of utmost importance to promote competition in public procurement within this sector.

Keywords: *Fuels and lubricants, public procurement, Benford´s Law, competence.*

Resumen

Este trabajo de investigación surge ante la necesidad que tiene el Paraguay de hacer un uso eficiente de los recursos utilizados en las compras públicas en el rubro de combustibles y lubricantes, cuya importancia radica en que, al ser Paraguay un país mediterráneo e importador neto de combustibles, requiere de grandes sumas para su adquisición, que luego serán utilizados por diferentes instituciones públicas tales como salud y seguridad interna para el correcto funcionamiento de sus servicios. El método utilizado es la Ley de Benford o también conocida, como la Ley del Primer Dígito que, si bien carece de un enfoque teórico, tiene una importancia estratégica desde el punto de vista empírico. El objetivo principal de este trabajo es el de determinar la existencia de posibles anomalías o irregularidades en las compras públicas de combustibles y lubricantes en Paraguay durante el período 2014 – 2023. Los resultados indican anomalías en la frecuencia de los dígitos en las tres variables analizadas: *montos adjudicados*, *cantidad de contratos u oferentes adjudicados por proceso* y *cantidad de oferentes presentados por proceso*. Se concluye que, ante estos resultados, es de altísima importancia promover la competencia en las compras públicas del rubro de estudio.

Palabras clave: *Combustibles y lubricantes, compras públicas, Ley de Benford, competencia.*

Correspondence: luis.rios@uc.edu.py
Article received: October 25, 2024. Accepted for publication: February 25, 2025. Published: April 30, 2025.
Conflicts of interest: None..
Funding source: None.
Responsible editor:: Herib Caballero Campos. Universidad Americana. Asunción, Paraguay.

Website: http://revistacientifica.uamericana.edu.py/index.php/academo/

Introduction

One of the economic policy instruments available to governments through their various institutions across the national territory—aimed at meeting their needs and/or stimulating the economy—is public procurement of goods and services supplied by private companies in an open, competitive environment subject to high standards of transparency (National Competition Advocacy Office, 2022).

During 2023, the total number of procurement processes announced (whether awarded, under evaluation, or pending opening) reached 8,533, representing 6.5% fewer than in 2022. In monetary terms, this corresponded to Gs. 22.87 trillion (USD 3.176 billion[2]), which is 0.24% higher than the amount recorded in the previous year (DNCP, 2024).

In general terms, public procurement has played a fundamental role in the country's development, as evidenced by its share in the National General Budget (PGN), which decreased from 25.6% of the PGN and 8.6% of GDP in 2022 to 22.8% and 7.9%, respectively, in 2023 (MEF, 2024; DNCP, 2024).

The procurement of fuels and lubricants represents an important category within public contracting. Given their essential nature and the fact that Paraguay is a net importer of fuels and lubricants, these acquisitions demand significant resources from the national budget (Bermo, 2025). However, it could be observed that regulatory bodies may lack sufficient control mechanisms to ensure the minimum enforcement of market laws within this particular area of procurement. If this were the case, the lack of oversight could result in inefficient use of public resources, suspicions of corruption, low transparency, poor-quality products, and above-market prices (Loiola, 2021).

Furthermore, the possibility that non-institutional factors might hinder stronger and fairer competition in bidding processes for fuel and lubricant supply should be carefully examined. Such factors may include excessive direct contracting, subcontracting, or a notable market concentration in awards and contracts, among others. These signs reinforce the suspicion of risks that could weaken fair competition (Calle García et al., 2023).

Competition fosters a level playing field among suppliers, enabling the government to access higher-quality products at more competitive prices. Moreover, competition prevents market concentration in the hands of a few actors, reducing the risk of monopolistic practices or collusion among suppliers (Nicholson, 2005). The presence of multiple bidders also promotes innovation and continuous improvement, which can lead to better technological solutions and more efficient products (Barack et al., 2017). Therefore, healthy competition in public procurement contributes to more efficient use of public resources, optimizing costs and ensuring that acquired goods meet required needs and standards (Pepall, 2006).

The objective of this study is to determine the existence of possible anomalies or irregularities in public procurement of fuels and lubricants in Paraguay during the period 2014–2023.

Methodology

This study is a quantitative documentary and bibliographic research, applying Benford's Law, or the First-Digit Law, to identify potential anomalies or irregularities in the following variables:

- Awarded amounts
- Number of contracts or awarded suppliers per process
- Number of suppliers participating per process during the period 2014–2023 (National Competition Advocacy Office, 2022).

The materials used in this research include statistical data from official sources such as the National Directorate of Public Procurement (DNCP) and bibliographic-documentary texts. Benford's Law[3] (also known as the First-Digit Law) is used as an empirical mathematical tool to detect anomalies or irregularities in numerical data distributions. According to this law,

[2] Rate of Exchange Gs./USD 7,200.

[3] Although it bears the name of physicist Frank Benford (1883–1948), who provided its mathematical formulation, it was astronomer Simon Newcomb (1835–1909) who first observed this phenomenon.

there exists a statistically regular distribution of the first digits (1–9). Empirical evidence shows that the number 1 has the highest probability of appearing as the first digit (approximately 30%), followed by 2 (around 18%), with the probability decreasing progressively up to 9 (Table 1) (Berger & Hill, 2011; Caputi Zunini, 2016; Carrera, 2015).

Based on empirical distribution, Benford's Law defines the following mathematical function to describe this behavior:

$$P(n) = \log_{10}(1 + \frac{1}{n}) = \log_{10}(1 + n) - \log_{10}(n) \quad \forall\, n = 1,2,3,\dots,9$$

Using this function, the theoretical distribution of digits can be constructed.

Table 1. Theoretical Distribution of Digits under Benford's Law.

Digit	Theoretical Frequency
1	30,10%
2	17,61%
3	12,49%
4	9,69%
5	7,92%
6	6,69%
7	5,80%
8	5,12%
9	4,58%

One of the most important applications of Benford's Law is that, based on this theoretical distribution, it allows us to compare the observed distribution of the digits of any variable and detect anomalous patterns, which would analytically indicate irregular or suspicious elements (Cabeza García, 2021).

In this study, the First-Digit Law was applied to three relevant variables in the award processes: the awarded amounts, the number of awarded contracts, and the number of bidders participating in each process.

Results

First-Digit Law by Awarded Amount

During the period 2014–2023, the first digits of the awarded amounts in each contract showed the following observed frequencies (Table 2).

Table 2. Benford's Law according to awarded amounts, 2014–2023

Digit	Observed Frequency
1	47,77%
2	12,69%
3	7,58%
4	7,24%
5	7,75%
6	5,62%
7	4,13%
8	3,83%
9	5,41%

Source: Author's own elaboration based on DNCP data.

Based on Table 2, a comparative visualization can be made between the distribution of the first digits observed and the theoretical distribution according to Benford's Law.

Figure 1 shows the distribution of the theoretical and observed frequencies of the digits corresponding to the analyzed variable. Comparatively, it can be seen that the digits of the awarded amounts do not statistically follow the same theoretical distribution, from which it can be inferred that there are suspicions of anomalies in the processes involving this particular variable.

Individually, digit 1 appears approximately 17% higher than expected under Benford's Law, while digit 3 appears about 5% lower than the theoretical distribution predicts. For the remaining digits, no major anomalies are observed.

In financial terms, the presence of digit 1 is usually associated with large amounts. Therefore, it can be inferred that in most contracts for the acquisition of fuels and lubricants, extraordinarily high amounts are involved, as indicated by the First-Digit Law.

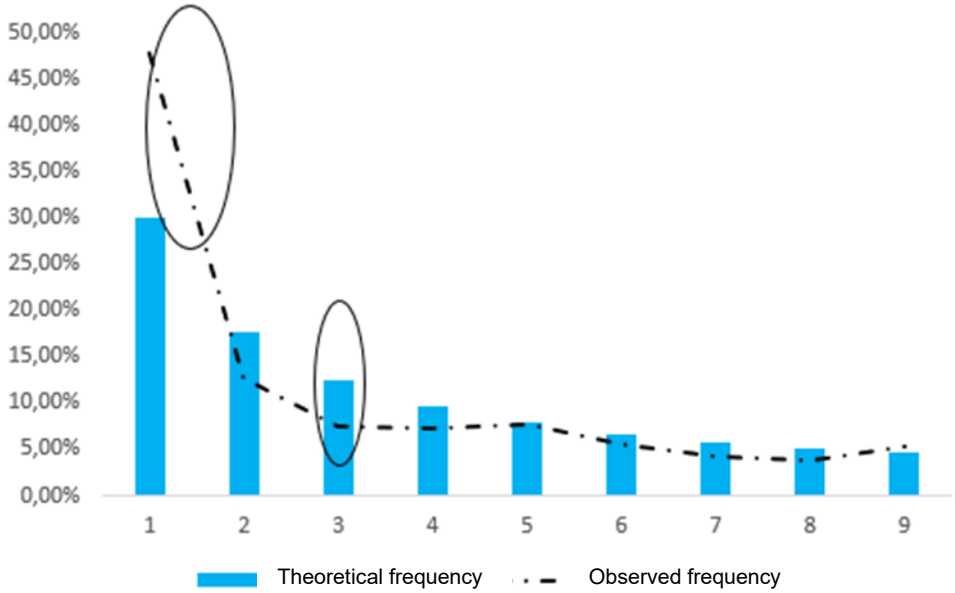

Figure 1. Benford's Law according to awarded amounts, 2014–2023. **Source:** Author's own elaboration based on DNCP data.

First-Digit Law by Number of Awarded Contracts or Bidders per Process

During the period 2014–2023, the first digits of the *number of awarded contracts or awarded bidders* in each process showed the following observed frequencies (Table 3).

Table 3. Benford's Law according to awarded contracts, 2014–2023.

Digit	Observed Frequency
1	92,96%
2	5,78%
3	0,76%
4	0,23%
5	0,14%
6	0,05%
7	0%
8	0%
9	0,09%

Source: Author's own elaboration based on DNCP data.

From table 3, it can be observed that digit 1 shows an exceptionally high frequency, while the frequencies of the other digits are considerably low.

Figure 2 presents the distribution of the theoretical and observed frequencies of the digits for this variable. Comparatively, it can be seen that the digits corresponding to the awarded contracts (or awarded bidders) per process do not statistically follow the same theoretical distribution, suggesting the presence of potential irregularities

Individually, digit 1 appears approximately 60% higher than expected under Benford's Law, while the remaining digits fall significantly below the theoretical values.

From this, it can be inferred that the majority of awarded contracts were likely made under the Direct Contracting modality (Figure 3).

Finally, it is confirmed that approximately 57% of the contracts awarded during the period 2013–2024 were under the direct contracting modality (single bidder), followed by competitive bidding with 22%, national public tender (LPN) with 13%, and other modalities with 8.3%.

The predominance of direct contracting in these calls is undeniably concerning, as it implies low levels of competition in the tenders related to this sector (fuels and lubricants).

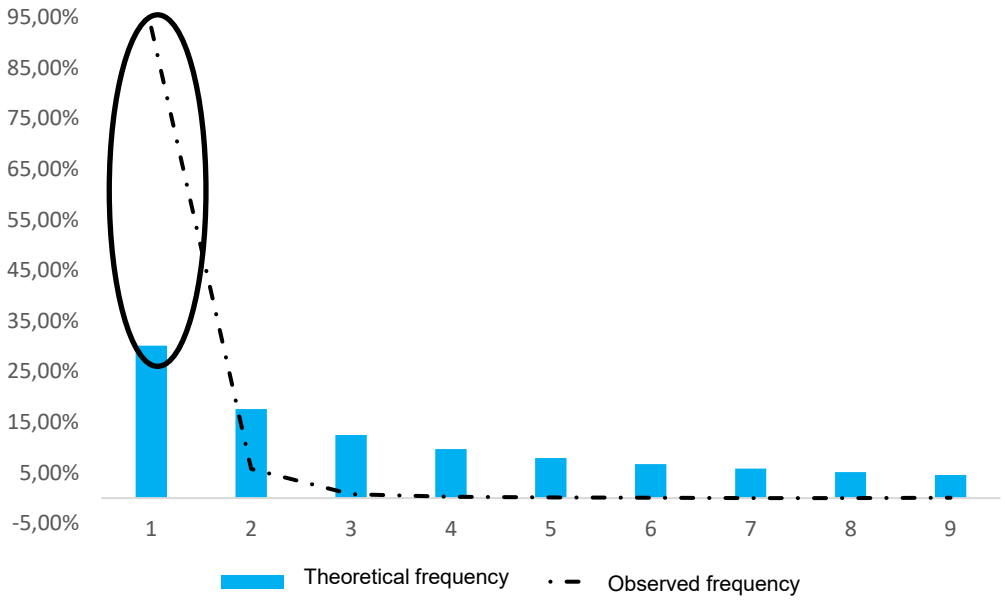

Figure 2. Benford's Law according to the number of awarded contracts per process, 2014–2023. **Source:** Author's own elaboration based on DNCP data.

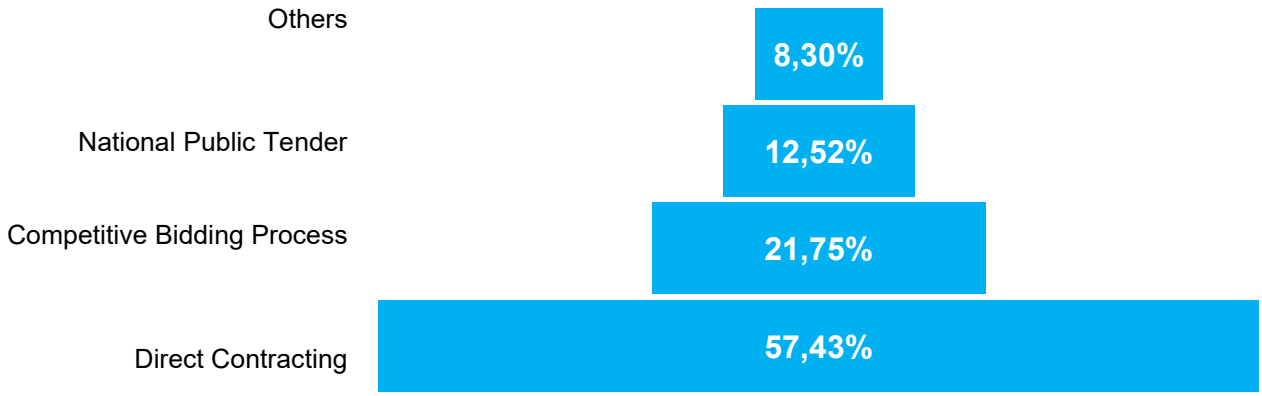

Figure 3. Number of contracts awarded by modality. Period 2014–2023. **Source:** Own elaboration based on DNCP data.

First Digit Law according to the number of bidders per process

During the period 2014–2023, the first digits of the number of bidders presented in each process showed the following observed frequencies (Table 4).

From this table, it can be observed that digit 1 has an impressively high frequency, while the frequencies of the other digits are considerably low

Table 4. Benford's Law according to awarded contracts. Period 2014–2023.

Digit	Observed Frequency
1	73,15%
2	10,11%
3	7,79%
4	1,82%
5	0,93%
6	0,65%
7	0,09%
8	0,05%
9	0,09%

Source: Own elaboration based on DNCP data.

Figure 4 shows the distribution of the theoretical and observed frequencies of the digits for the variable under analysis. Comparatively, it is evident that the digits corresponding to the number of bidders per process do not statistically follow the theoretical distribution, suggesting the presence of potential irregularities.

Individually, the digit 1 appears approximately 40% higher than expected under Benford's Law, while the other digits fall well below expectations.This result is very consistent with the previous finding—namely, that the predominance of direct contracting is explained by the presence of single bidders, since it is technically unreasonable to call for tenders when there is only one supplier in the market. However, this reinforces the idea of extremely low competition in public procurement within the sector under study.

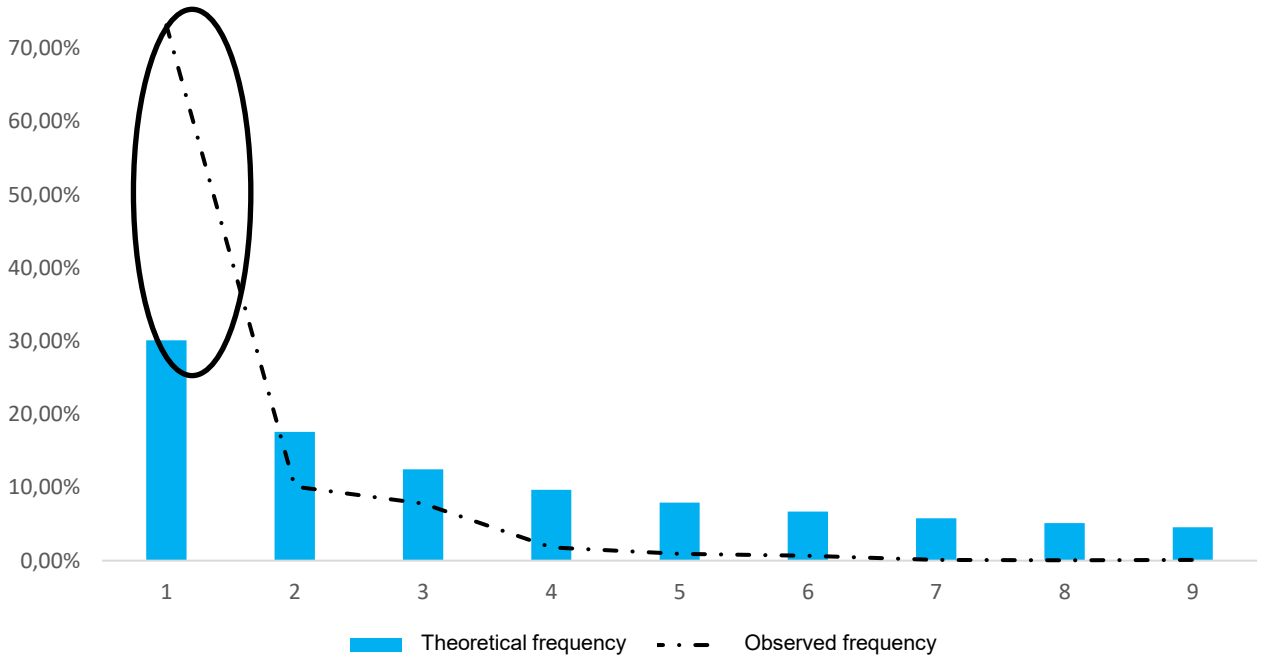

Figure 4. Benford's Law according to the number of contracts awarded in each process. Period 2014–2023. **Source:** Own elaboration based on DNCP data.

Discussion

The results obtained under Benford's Law allow us to identify anomalies and irregularities in the data that could generate inefficiencies in public procurement processes for fuels and lubricants in the Paraguayan market.

First, considering the amounts awarded in the tenders, slight discrepancies can be observed between the theoretical and the observed distribution of digits 1 and 3, which financially implies extraordinarily high awarded amounts. Although, under the First-Digit Law, this represents an anomaly, it does not necessarily indicate irregularities in the processes per se.

Second, considering the number of contracts or awarded bidders per process in the tenders, very large discrepancies can be observed between the theoretical and observed distribution of digit 1 in particular, with the observed value being 60% higher than the theoretical one. Analytically, it can be deduced that most of the contracts were awarded under the modality of direct contracting, which represents approximately 57% of all awarded contracts.

Third, considering the number of bidders participating per process in the tenders, very large discrepancies can also be observed between the theoretical and observed distribution of digit 1, with the observed value being 30% higher than the theoretical one.

Analytically, this strengthens our hypothesis of low market competition.

The fundamental importance of this study lies in attempting to identify potential irregularities that could help improve procurement processes and, consequently, enhance efficiency in both timing and implementation.

In conclusion, it is essential that procurement processes are not only transparent but also as competitive as possible—that is, they should promote firm participation in tenders in order to ensure that products are of good quality and reasonably priced, thereby using public resources and taxpayers' money responsibly.

The final recommendation for future studies would be to identify exactly which factors limit competition in tenders, such as bureaucracy, delayed payments by the State, and the presence of unnecessary requirements that act as barriers, among others.

References

Barack, R., Mitchell, W., Vidal, E., & Schulman, K. (2017). Pharmaceutical M&A activity: Effects on prices, innovation and competition. *Loyola University Chicago Law Journal*, 48,787-819. https://scholarship.law.duke.edu/cgi/viewcontent.cgi?article=6441&context=faculty_scholarship

Bermo, S. (4 de febrero de 2025). Los desafíos del mercado de combustibles en Paraguay: desde la bajante del río hasta los ajustes de precio. Surtidores Latam Paraguay. https://surtidoreslatam.com/economia-los-desafios-del-mercado-de-combustibles-en-paraguay-desde-la-bajante-del-rio-hasta-los-ajustes-de-precio/

Berger, A., & Hill, T. P. (2011). A basic theory of Benford's Law. Project Euclid. *Probability Surveys*, 8, 1-126. https://core.ac.uk/download/pdf/19161099.pdf.

Cabeza García, M. (2021). Aplicación de la ley de Benford en la detección de fraudes. *Universidad y Sociedad*, 13(4). http://scielo.sld.cu/scielo.php?script=sci_arttext&pid=S2218-36202021000400461.

Calle García, A. J., Quiñónez Cercado, M. P., Espinoza Tuárez, Y. M. (2023). Auditorías a la contratación pública de combustibles y lubricantes en las universidades de Ecuador. *Ciencia y Desarrollo,* 26(3), 109-118. https://dialnet.unirioja.es/servlet/articulo?codigo=9238587.

Caputi Zunini, M. (2016). *Ley de Benford*. Instituto de Perfeccionamiento y Estudios CFE/ANEP-UDELAR. https://repositorio.cfe.edu.uy/bitstream/handle/123456789/247/Caputi%20Zunini%2C%20Maria%20%2C%20Ley.pdf?sequence=1&isAllowed=y.

Carrera, C. (2015). La ley de Benford y su aplicación a la economía. *Revista Moneda, Banco. Central de Reserva del Perú*, 47-49. https://www.bcrp.gob.pe/docs/Publicaciones/Revista-Moneda/moneda-165/moneda-165-11.pdf.

DNCP. (2024). Dirección de Contrataciones Públicas Adjudicaciones. https://www.contrataciones.gov.py/dncp/estadisticas/adjudicaciones/

Intendencia Nacional de Abogacía de la Competencia. (2022). *Estudio de Mercado del Sistema Nacional de Contrataciones Públicas.* Dirección Nacional de Estudios de Mercado.

Loiola, A. D. (2021). *Public Procurement in Brazil: Evidence of frauds using the newcomb-benford law*. Universidade Federal da Bahia, Escola de Administração. https://www.researchgate.net/publication/357622565_PUBLIC_PROCUREMENT_IN_BRAZIL_EVIDENCE_OF_FRAUDS_USING_THE_NEWCOMB-BENFORD_LAW.

MEF. (2024). *Ministerio de Economía y Finanzas*. https://www.mef.gov.py/situfin

Nicholson, W. (2005). Teoría microeconómica: Principios básicos y ampliaciones. 9 ed. Cengage Learning.

Pepall, R. R. (2006). Organización industrial: Teoría y práctica contemporáneas. 3 ed. Thompson Learning.